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ANOMALOUS CHOKING CONDITIONS RELATED TO TWO-PHASE PRESSURE DROP COMPUTATIONS

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Abstract—Unacceptable anomalous upstream choking conditions are shown to occur at low qualities when using the most commonly applied engineering model for computing two-phase (liquid–vapor) pressure drop. The direct relation between this condition and the mathematical prediction of a maximum value of the critical mass velocity in regions where quality approaches zero, is noted. Although computations for vapor–liquid sodium flows are emphasized, it is noted that similar difficulties will also exist with water flows. Upstream choking is shown to result from use of four different representations of the Lockheart–Martinelli vapor volume fraction correlation as well as from three other void correlations, two of which are specifically suggested for application to liquid metals.

NOMENCLATURE

D, diameter of duct [ft];
g, acceleration of gravity [ft/s^2];
G, mass velocity [lb$_m$/h ft^2];
G_c, critical mass velocity [lb$_m$/h ft^2];
h, enthalpy [Btu/lb$_m$];
h_0, stagnation enthalpy [Btu/lb$_m$];
h_f, enthalpy of saturated liquid [Btu/lb$_m$];
h_{fg}, latent heat of vaporization [Btu/lb$_m$];
k, slip ratio [dimensionless];
L, axial coordinate [ft];
p, pressure [psia];
p_E, duct exit pressure [psia];
p_p, exit plenum pressure [psia];
p_{sat}, saturation pressure at $h_0 = h_f$ [psia];
v_g, specific volume of vapor [ft^3/lb$_m$];
v_f, specific volume of liquid [ft^3/lb$_m$];
x, quality;
α, vapor volume fraction (void fraction);
τ_w, wall shear of two-phase flow [lb$_f$/ft^2];
τ_{w0}, wall shear of all liquid flow at system G, [lb$_f$/ft^2];
ρ, average density [lb$_m$/ft^3].

INTRODUCTION

ENGINEERING pressure drop computations for the flow of two-phase (vapor–liquid) mixtures are usually, if not always, based on a one dimensional mathematical model under the assumptions of (1) thermodynamic equilibrium, and (2) the vapor volume fraction, α, a known function of fluid physical properties and vapor weight fraction (quality), x. This model will be referred to as the "engineering computational model." It is important to note that this model is widely used at present to predict two-phase pressure drop in spite of questions that have been raised with regard to its validity [1]. In fact, except for variations in the methods used to predict α and frictional pressure drop, there do not appear to be any alternates currently in use for engineering applications. The Lockhart–Martinelli [2] correlations for α and frictional pressure drop are probably the most popular.

In most applications involving vapor–liquid flows, the fluid enters the duct as a liquid and vaporization occurs at some location downstream of the inlet. As a result, the engineering computational model must be applied to regions of incipient boiling and, of course, very small qualities. Satisfactory accuracy has been demonstrated by comparison with experimental data

only for cases in which the axial gradients of quality, vapor volume fraction, and vapor density were small [1, 3]. There is, however, current interest in systems in which these axial gradients are very large, for example, in the Liquid–Metal Fast Breeder Reactor safety program with low quality vapor–liquid sodium flows at pressures above 20 psia. Although qualities are of extremely small magnitude in this type of application, vapor volume fractions are relatively large.

Recently, Fauske and Grolmes [4] obtained experimental choking flow rate (critical mass velocity) and pressure drop data for adiabatic, low quality, two-phase sodium flow through tubes with pressures ranging from 1 to 8 psia. They found that adequate predictions could be made of both the critical mass velocities and the pressure distributions along the tube. Their computations were based on the engineering computational model with α obtained from an interpolation routine representing the Lockhart–Martinelli correlation. The momentum equation in differential form was integrated from the tube outlet in the upstream direction. However, for some cases, as quality approached zero upstream of the outlet, the model predicted an infinite pressure gradient. This situation may be interpreted as equivalent to an apparent upstream choking condition. As a result, the same computational model could not be used to complete the upstream integration to zero quality.

The calculations of Fauske and Gromles predicted choking at both the tube outlet and at positions upstream very close to zero quality. This result is equivalent to a prediction of a maximum critical mass velocity as quality approaches zero, since $G = G_c$ at both apparent choking locations, but $G < G_c$ everywhere between these two locations. In fact, calculated maximum critical mass velocities have been noted for water flows by Levy [5] using his correlation for α, by Nahavandi and Rashevsky [6] using both the Lockhart–Martinelli and modified Armand (as cited by Fauske [7]) correlations for α, and by Henry [8] based on Moody's [9] method of computing choked flows. The occurrence of these maxima were not previously related to anomalous upstream choking conditions that may occur in pressure drop computations.

The purpose of this article is to suggest that anomalous upstream choking conditions as quality approaches zero, can occur generally with the engineering computational model and that its occurrence is a result of inaccurate assumptions implicit in the model. Specific attention is given to liquid metals with emphasis on the relationship of the occurrence of an upstream choking condition to the prediction of a maximum G_c. The engineering computational model was applied to sodium flows at higher pressures than considered by Fauske and Grolmes [4, 10] and with other methods of computing α. Included were correlations for α specifically suggested for use with liquid metals, and various alternate mathematical representations of the Lockhart–Martinelli correlation.

MATHEMATICAL MODEL

The engineering computational model as applied to the adiabatic upward flow of a liquid–vapor mixture through a constant area duct may be written as

$$\frac{dL}{dp} = \frac{1 + G^2 (dv_M/dp)}{F} \tag{1}$$

and

$$h_0 = h_f + xh_{fg} + \tfrac{1}{2}G^2 v_k^2 \tag{2}$$

where

$$v_M = \frac{x^2 v_g}{\alpha} + \frac{(1-x)^2 v_f}{1-\alpha}; \qquad F = \frac{4\tau_w}{D} + \rho g$$

$$v_k^2 = \frac{x^3}{\alpha^2} v_g^2 + \frac{(1-x)^3}{(1-\alpha)^2} v_f^2; \qquad \tau_w = \tau_{w0} \left(\frac{1-x}{1-\alpha}\right)^2.$$

Referring to equation (1), L is the axial coordinate defined positive in the upstream direction. The expression for τ_w as given above is an

accurate representation of the Lockhart–Martinelli method of computing two-phase frictional wall shear stress suggested by Lottes [11]. The principal point here is the choice of the function $\alpha = \alpha(p, x)$ necessary to complete the model. The Fauske–Gromles investigations indicate that, except when an anomalous upstream choking occurs, the use of the Lockhart–Martinelli correlation for $\alpha(p, x)$ in the above equations results in fairly good agreement with experimental data for sodium. Thus, alternate choices employed for $\alpha(p, x)$ should give α predictions reasonably close to those of the Lockhart–Martinelli correlation. A choking condition is indicated when $-G^2\, dv_M/dp = 1$, and the influence of the vapor volume fraction correlation on this condition is, of course, pertinent.

With the stagnation enthalpy, $h_0 (h_0 \geqslant h_f)$, outlet plenum pressure, p_p, and mass velocity, G, specified, the right hand side of equation (1) is a function of pressure only, and numerical integration with pressure as the independent variable is straight forward for various choices of $\alpha = \alpha(p, x)$. When outlet choking did not occur [i.e. when $-G^2\,(dv_M/dp) < 1$], the tube exit pressure, p_E, was taken equal to p_p. When outlet choking occurred, the exit pressure ($p_E > p_p$) was evaluated by satisfying the condition $-G^2\,(dv_M/dp) = 1$. In either case, integration of equation (1) proceeded upstream using the exit pressure as the initial condition. The intent was to proceed with the upstream integration to the axial location in the duct where quality was equal to zero. The relations of Golden and Tokar [12] were used for the physical properties of sodium. Computations were performed with a CDC-3600 digital computer.

RESULTS

Illustrative examples of the results obtained for a low quality, adiabatic, sodium flow through a circular tube into an outlet plenum at 20 psia are shown in Fig. 1. The stagnation enthalpy, h_0, is 700 Btu/lb$_m$, which corresponds to a saturation pressure of 34·1 psia, and to a quality of 0·023 in the outlet plenum. Computations were based on an empirical relationship representing the Lockhart–Martinelli α correlation given by Wallis, equation (3·32) of [1]. The Blasius friction factor relationship was used to determine τ_{w0}.

Referring to the curve of Fig. 1 for which the mass velocity is 3·1 $\times$ 10^6 lb$_m$/(h) (ft)^2, the occurrence of outlet choking results in a corresponding exit pressure of 26 psia, and an anomalous upstream choking condition was obtained at an axial position three tube diameters from the outlet. At this position $\alpha = 0{\cdot}5$, $x = 0{\cdot}0025$, and the local static pressure was approximately 2 psi less than the saturation pressure at $h_f = h_0$.

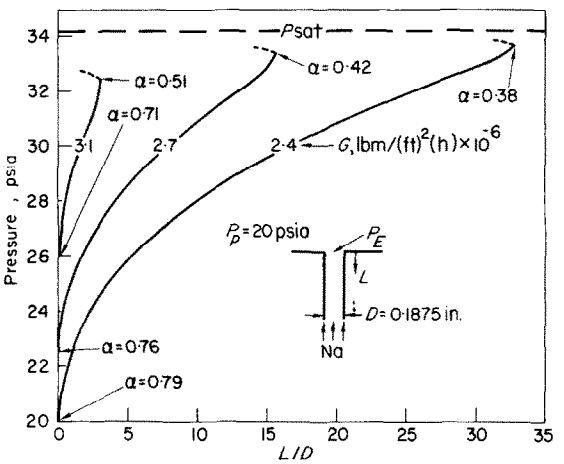

FIG. 1. Illustrative results of anomalous upstream choking.

Computations beyond this point resulted in negative values of the computed length increment as pressure increased and is represented by the dashed portion of the pressure curve. Thus, it was not possible to integrate equation (1) upstream to the location where $x = 0$ and $\alpha = 0$, the point of incipient boiling.

It should also be noted that integration downstream from the pressure corresponding to the saturation pressure at $h_f = h_0$ would not be possible either since dp/dL would be negative there. The effect would be the prediction of zero boiling length, a result entirely different from the upstream integration prediction.

Computed pressure vs. length curves for two lower values of mass velocity are also shown in Fig. 1. As indicated, anomalous upstream choking conditions were obtained for these cases also. Note that for the lowest flow rate illustrated, outlet choking did not occur; the tube exit pressure is equal to the plenum pressure. As the flow rate decreases, the static pressure at the point of upstream choking approaches the saturation pressure at $h_0 = h_f$, and for sufficiently small flow rates, an anomalous upstream choking condition would not occur.

A local critical mass velocity can be obtained from equation (1) as $G_c^{-2} = -\mathrm{d}v_M/\mathrm{d}p$ which implies that the momentum term is analogous to the square of the Mach number of single-phase compressible flow and is equal to unity when choking occurs. Thus, for the two larger mass velocity curves of Fig. 1, the analogous Mach number is unity at the outlet, initially decreases with increasing pressure, reaches a minimum, and then increases to unity again at the point of upstream choking. Alternatively, G_c initially increases, reaches a maximum, and then decreases as $x \to 0$.

The results shown in Fig. 1 are typical of those obtained for stagnation enthalpies corresponding to sodium saturation pressures ranging from 1 psia to 73·5 psia. The use of other vapor volume fraction correlations was also explored primarily for the purpose of determining which of them would result in the equivalent of a maximum G_c as $x \to 0$. Since the kinetic energy term of equation (2) was nearly negligible compared to h_0, the computations of G_c were based on the approximation $G_c^{-2} \simeq (\partial v_M/\partial P)_h$ with $h = h_f + xh_{fg}$.

Several different mathematical representations of the Lockhart–Martinelli correlation were investigated. Included were the following: (1) a theoretical relationship due to Wallis, equation (11.35) of [1]; (2) an empirical relationship given by MacFarlane, equation (16) of [13]; (3) an empirical relationship given by Hewitt *et al.*, equation (24) of [14]. The vapor volume fractions as a function of quality predicted by these Lockhart–Martinelli representations compared favorably with each other and with the Wallis empirical relation—equation (3.32) of [1]—used in Fig. 1 for $h_0 = 700$ Btu/lb$_m$, over the quality range of interest ($0{\cdot}001 < x < 0{\cdot}02$). The relation $k = (v_g/v_f)^{0{\cdot}5}$ which is the basis for Fauske's method [7] of computing G_c, did not agree at all with Lockhart–Martinelli.

The computed values of G_c based on the four representations of the Lockhart–Martinelli α correlation are shown in Fig. 2; all result in a maximum value. As found also by others [5–7], the use of $k = (v_g/v_f)^{0{\cdot}5}$ for computing G_c eliminated the occurrence of a maximum value.

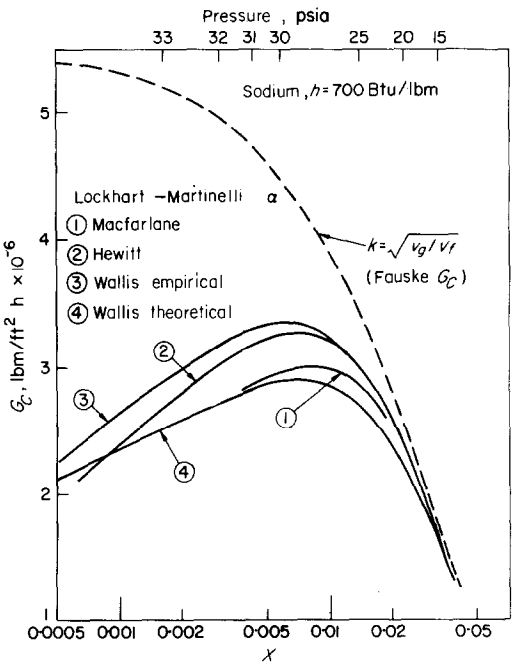

FIG. 2. Critical mass velocity predictions based on the Lockhart–Martinelli α correlation.

It may be inferred from the figure that different mathematical representations of the Lockhart–Martinelli correlation can have a very significant effect on the predicted pressure and vapor volume fraction distributions of Fig. 1. For example, if the Wallis empirical representation

were replaced by the MacFarlane or Wallis theoretical relationships for the case of Fig. 1 with $G = 3{\cdot}1 \times 10^6$ lb$_m$/(ft)^2 (h), outlet choking with complete suppression of vaporization in the tube ($p_E = p_{sat}, \alpha = 0$) would be predicted. This difference would occur because the predicted maximum values of G_c are less than the actual mass velocity, G.

Other vapor volume fraction correlations investigated included the following: (1) Baroczy's correlation [15, 16]; (3) Balzhiser's correlation as modified by MacFarlane, equation (A18) of [13]; (3) Levy's [3] correlation, $k = (\alpha v_g/2v_f)^{0{\cdot}5}$.

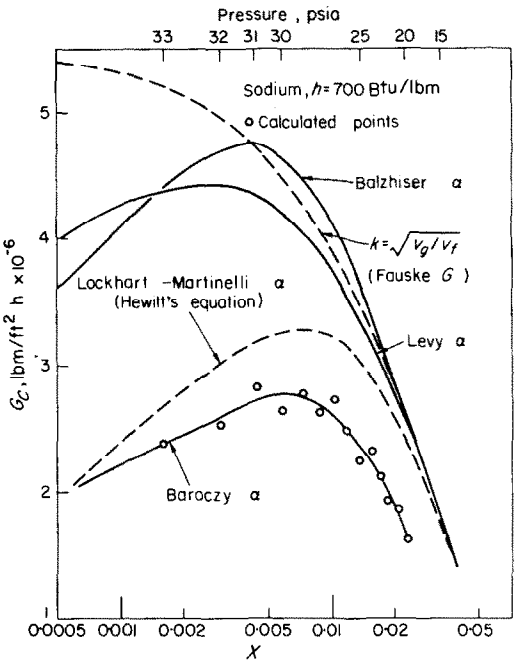

FIG. 3. Critical mass velocity predictions based on Baroczy, Levy and Balzhiser α correlations.

The Baroczy and Balzhiser correlations are empirical, based on liquid-metal data. Levy's equation, like the Wallis theoretical representation for the Lockhart–Martinelli correlation, was derived from a simplified theory which did not intend to account for large axial gradients. Note that Levy's equation implies that $k \to 0$ as $\alpha \to 0$. However, maximum values of G_c were obtained well before k became less than unity with all α correlations used. It was found that Baroczy's α correlation yields results relatively close to the Lockhart–Martinelli predictions, but that the Levy and Balzhiser equations offer only a slight improvement over the use of $k = (v_g/v_f)^{0{\cdot}5}$. Computations of G_c corresponding to these α correlations are shown in Fig. 3. Maximum values of G_c result from all of the correlations except, of course, for the use of $k = (v_g/v_f)^{0{\cdot}5}$. Computations with the Baroczy correlation did not give completely consistent results as indicated by the scatter of the calculated points shown. Presumably, this effect was caused by the necessity to interpolate among Baroczy's tabulated values, and then to numerically differentiate quantities which depend on α.

Use of the more general slip ratio relationship $k = (v_g(v_f)^n$, was also explored. Although this relationship eliminated the problem of a maximum G_c for $0 \leqslant n \leqslant 1$,† it did not result in sufficiently improved predictions when compared to the Lockhart–Martinelli correlation.

CONCLUSIONS

Results for G_c at $h = 700$ Btu/lb$_m$ given in Figs. 2 and 3 are typical of those obtained in this investigation for sodium. Stagnation enthalpy ranged from 600 to 763 Btu/lb$_m$ and the pressure from 1 to 73·5 psia. Although the occurrence of a maximum G_c with the α correlations explored here and by others, which allows for the possibility of an anomalous upstream choking condition, is suspected to be true for all fluids below their thermodynamic critical pressures, this generalization has not yet been established.

There are two general views that may be taken regarding the above results. The first

† A reviewer of this article pointed out that Moody's [9] method of computing G_c, which also results in a maximum value [8], is based on $n = \frac{1}{3}$ and hence seems to contradict the result reported above. However, Moody's method of computing G_c is also based on use of v_k rather than v_M, and hence does not strictly apply to the above.

points to the fact that the engineering computational model represented by equations (1) and (2) and the Lockhart–Martinelli correlation for α satisfactorily predicts the Fauske–Grolmes choked flow sodium pressure data up to a location of apparent upstream choking. Why not, then, accept the model up to this location, and then change to another computational method to complete the calculations? For example, in [4], all liquid flow was assumed to exist immediately upstream of the anomalous upstream choking point. In most cases, upstream choking occurs, if at all, at pressures very close to saturated conditions at $x = 0$. Thus, this point of view suggests, any reasonable computational technique applied to the region of upstream choking would not significantly affect overall pressure drop predictions in most applications.

The second point of view, favoured by the authors, suggests that the existence of a minimum in $-dv_M/dp$ for use with equation (1), or equivalently the existence of a maximum in G_c, is physically unacceptable. This view would be supported by analogy with single-phase compressible flow, and by noting that it would be possible to obtain entirely different results depending on whether integration of equation (1) proceeds in the upstream or downstream direction. There is some sodium G_c data [8] available in the low quality regime which do not exhibit a maximum lending further support. This second view additionally suggests that the assumptions implicit in the engineering computational model are too inaccurate for systems in which axial gradients are large, and the use of more sophisticated computational models should be explored.

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CONDITIONS ANORMALES D'OBSTRUCTION RELIÉES AU CALCUL DE CHUTE DE PRESSION DANS UN ÉCOULEMENT BIPHASIQUE

Résumé— Des conditions inacceptables et anormales d'obstruction apparaissent aux faibles valeurs de la qualité quand on utilise le modèle le plus couramment appliqué pour calculer la chute de pression en fluide biphasique (liquide–vapeur). On note la relation directe entre cette condition et la prévision mathématique d'une valeur maximale de la vitesse massique critique dans les régions où la qualité approche zéro. Bien que les calculs pour les écoulements vapeur–liquide de sodium sont développés on note que des difficultés semblables existent aussi avec les écoulements d'eau. L'obstruction en amont résulte de l'utilisation de quatre représentations différentes de la formule de Lockart–Martinelli relative à la fraction

volumique de vapeur aussi bien que de trois autres formules de vide dont deux d'entre elles sont spécifiquement suggégées pour l'application des métaux liquides.

ANOMALE DROSSELBEDINGUNGEN BEI ZWEIPHASEN-DRUCKABFALLRECHNUNGEN

Zusammenfassung—Bei der Berechnung des Zweiphasendruckabfalls mit Hilfe der allgemein angewandten Berechnungsmodelle zeigen sich bei geringen Qualitäten unannehmbare und abnorme Drosselbedingungen stromaufwärts. Die direkte Beziehung zwischen diesem Zustand und der mathematischen Berechnung des grössten Wertes der kritischen Massenstromdichte für Qualitäten nahe null wird aufgezeigt. Obwohl die Berechnungen für Natrium- Natrium-Dampfströmungen bevorzugt wurden, wird darauf hingewiesen, dass ähnliche Schwierigkeiten auch bei Wasserströmung auftreten. Die Drosselbedingungen stromaufwärts ergeben sich sowohl aus der Anwendung von 4 verschiedenen Darstellungen der Gleichung für den Dampfanteil von Lockhart-Martinelli als auch bei der Verwendung von 3 anderen Gleichungen für den Leerraumanteil, wovon zwei besonders für die Anwendung bei flüssigen Metallen vorgeschlagen sind.

УСЛОВИЯ АНОМАЛЬНОГО ЗАПИРАНИЯ, СВЯЗАННЫЕ С РАСЧЕТАМИ ПЕРЕПАДА ДАВЛЕНИЯ

Аннотация—В работе показано, что условия аномального запирания в верхнем течении имеют место при низких значениях истинного весового паросодержания при использовании широко распространенной модели для расчета перепада давления в двухфазном потоке (жидкость-пар). Отмечено прямое соотношение между этим условием и математическим расчетом максимального значения критической массовой скорости на участках с истинным весовым паросодержанием, приближающемся к нулю. Хотя расчеты относятся, в основном, к двухфазным течениям натрия, замечено, что аналогичные трудности имеют место также для течений воды. Показано, что запирание в верхнем течении возникает в результате использования четырех различных представлений соотношения Локхарта-Мартинелли для относительного объема пара, а также трех других соотношений относительного объемного паросодержания, два их которых предлагаются для жидких металлов.